DESCENT COHOMOLOGY AND CORINGS

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ABSTRACT. A coring approach to non-Abelian descent cohomology of [P. Nuss & M. Wambst, *Non-Abelian Hopf cohomology*, Preprint arXiv:math.KT/0511712, (2005)] is described and a definition of a Galois cohomology for partial group actions is proposed.

1. Introduction

1.1. **Motivation and aims.** In a recent paper [13] Nuss and Wambst have introduced a non-Abelian descent cohomology for Hopf modules and related it to classes of twisted forms of modules corresponding to a faithfully flat Hopf-Galois extension. Hopf modules can be understood as a special class of *entwined modules* and hence comodules of a coring [2]. The aim of this note is to show how the descent cohomology introduced in [13] fits into recent developments in the descent theory for corings [10], [4], [6]. In particular we construct the zeroth and first descent cohomology sets for a coring with values in a comodule and relate it to isomorphism classes of module-twisted forms. We then use this general framework to propose a definition of a non-Abelian Galois cohomology for idempotent partial Galois actions on non-commutative rings introduced in [5].

1.2. **Notation and conventions.** We work over a commutative associative ring k with unit. All algebras are associative, unital and over k. The identity map in a k-module M is denoted by M. Given an algebra A the coproduct in an A-coring $\mathfrak{C}$ is denoted by $\Delta_{\mathfrak{C}}$ and the counit by $\varepsilon_{\mathfrak{C}}$. A (fixed) coaction in a right $\mathfrak{C}$-comodule M is denoted by ϱ^M. $\mathrm{Hom}^{\mathfrak{C}}(-,-)$, $\mathrm{End}^{\mathfrak{C}}(-)$ and $\mathrm{Aut}^{\mathfrak{C}}(-)$ denote the homomorphisms, endomorphisms and automorphisms of right $\mathfrak{C}$-comodules, respectively. $\mathrm{End}^{\mathfrak{C}}(M)$ is a ring with the product given by the composition, M is a left $\mathrm{End}^{\mathfrak{C}}(M)$-module with the product given by evaluation and $\mathrm{Hom}^{\mathfrak{C}}(M,N)$ is a right $\mathrm{End}^{\mathfrak{C}}(M)$-module with the product given by composition.

For an algebra A, $\mathcal{G}(A)$ denotes the group of units in A, and for an A-coring $\mathfrak{C}$, $\mathcal{G}(\mathfrak{C})$ is the set of grouplike elements of $\mathfrak{C}$, i.e. elements $g \in G$ such that $\Delta_{\mathfrak{C}}(g) = g\otimes_A g$ and $\varepsilon_{\mathfrak{C}}(g) = 1$. $\mathcal{G}(\mathfrak{C})$ is a right $\mathcal{G}(A)$-set with the action given by the conjugation $g \cdot u := u^{-1}gu$, for all $u \in \mathcal{G}(A)$ and $g \in \mathcal{G}(\mathfrak{C})$.

More details on corings and comodules can be found in [3].

2. Descent and partial Galois cohomologies

2.1. **Construction of descent cohomology sets.** Given an A-coring $\mathfrak{C}$ and a right $\mathfrak{C}$-comodule M with fixed coaction $\varrho^M : M \to M\otimes_A\mathfrak{C}$, define the *zeroth descent cohomology group of* $\mathfrak{C}$ *with values in* M as the group of $\mathfrak{C}$-comodule automorphisms,

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i.e.

$$D^0(\mathfrak{C}, M) := \mathrm{Aut}^{\mathfrak{C}}(M).$$

Any isomorphism of right $\mathfrak{C}$-comodules, $f : \widetilde{M} \to M$, induces an isomorphism of cohomology groups

$$f^* : D^0(\mathfrak{C}, M) \to D^0(\mathfrak{C}, \widetilde{M}), \qquad \alpha \mapsto f^{-1} \circ \alpha \circ f.$$

The set $Z^1(\mathfrak{C}, M)$ of *descent 1-cocycles on $\mathfrak{C}$ with values in M* is defined as a set of all $\mathfrak{C}$-coactions $F : M \to M \otimes_A \mathfrak{C}$. Since M comes equipped with the right coaction ϱ^M, $Z^1(\mathfrak{C}, M)$ is a pointed set with a distinguished point ϱ^M.

Note that for the Sweedler coring $A \otimes_B A$ associated to a ring extension $B \to A$, $Z^1(A \otimes_B A, M)$ is the set of *non-commutative descent data* on M (cf. [3, Section 25.4]). This motivates the name *descent cocycles*.

Lemma 2.1. *Let $M, \widetilde{M}$ be right $\mathfrak{C}$-comodules. Any right A-linear isomorphism $f : \widetilde{M} \to M$ induces a bijection $f^* : Z^1(\mathfrak{C}, M) \to Z^1(\mathfrak{C}, \widetilde{M})$ defined by*

$$f^*(F) := (f^{-1} \otimes_A \mathfrak{C}) \circ F \circ f.$$

The operation $(-)^*$ maps the identity map into the identity map and reverses the order of the composition, i.e., for all right A-module isomorphisms $f : \widetilde{M} \to N$, $g : N \to M$, $(g \circ f)^* = f^* \circ g^*$. Furthermore, if $f : \widetilde{M} \to M$ is an isomorphism of $\mathfrak{C}$-comodules, then f^* is an isomorphism of pointed sets.

Proof. If $F : M \to M \otimes_A \mathfrak{C}$ is a right coaction, and $f : \widetilde{M} \to M$ is a right A-linear isomorphism, then

$$\begin{aligned}
(f^*(F) \otimes_A \mathfrak{C}) \circ f^*(F) &= (f^{-1} \otimes_A \mathfrak{C} \otimes_A \mathfrak{C}) \circ (F \otimes_A \mathfrak{C}) \circ (f \otimes_A \mathfrak{C}) \circ (f^{-1} \otimes_A \mathfrak{C}) \circ F \circ f \\
&= (f^{-1} \otimes_A \mathfrak{C} \otimes_A \mathfrak{C}) \circ (M \otimes_A \Delta_{\mathfrak{C}}) \circ F \circ f \\
&= (\widetilde{M} \otimes_A \Delta_{\mathfrak{C}}) \circ (f^{-1} \otimes_A \mathfrak{C}) \circ F \circ f = (\widetilde{M} \otimes_A \Delta_{\mathfrak{C}}) \circ f^*(F),
\end{aligned}$$

where the second equality follows by the coassociativity of F. This proves that $f^*(F)$ is a coassociative coaction. The counitality of F immediately implies that also $f^*(F)$ is a counital map. Thus f^* is a well-defined map, and it is obviously a bijection as stated. The proofs of remaining statements are straightforward. □

Lemma 2.1 immediately implies that there is a right action of the A-linear automorphism group $\mathrm{Aut}_A(M)$ on $Z^1(\mathfrak{C}, M)$ given by

$$Z^1(\mathfrak{C}, M) \times \mathrm{Aut}_A(M) \to Z^1(\mathfrak{C}, M), \qquad (F, f) \mapsto f^*(F).$$

Definition 2.2. The *first descent cohomology set of $\mathfrak{C}$ with values in the $\mathfrak{C}$-comodule* M is defined as the quotient of $Z^1(\mathfrak{C}, M)$ by the action of $\mathrm{Aut}_A(M)$ and is denoted by $D^1(\mathfrak{C}, M)$.

Since $Z^1(\mathfrak{C}, M)$ is a pointed set, so is $D^1(\mathfrak{C}, M)$ with the class of ϱ^M as a distinguished point.

2.2. M**-torsors.** Given a right $\mathfrak{C}$-comodule M, an M*-torsor* is a triple (X, ϱ^X, β), where X is a right $\mathfrak{C}$-comodule with the coaction ϱ^X and $\beta : M \to X$ is an isomorphism of right A-modules. Two M-torsors (X, ϱ^X, β) and (Y, ϱ^Y, γ) are said to be *equivalent* if (X, ϱ^X) and (Y, ϱ^Y) are isomorphic as comodules. Equivalence classes of M-torsors are denoted by $\mathrm{Tors}(M)$. $\mathrm{Tors}(M)$ is a pointed set with the class of the M-torsor (M, ϱ^M, M) as a distinguished point. Since $\mathrm{Tors}(M)$ is a set of isomorphism classes of comodules which are isomorphic to M as modules, one obtains the following description of $\mathrm{Tors}(M)$.

Proposition 2.3. *For all $\mathfrak{C}$-comodules M, there is an isomorphism of pointed sets*

$$D^1(\mathfrak{C}, M) \simeq \mathrm{Tors}(M).$$

Proof. Explicitly, the isomorphism is constructed as follows. Given a coaction $F : M \to M\otimes_A\mathfrak{C}$, consider an M-torsor $T(F) := (M, F, M)$. Conversely, given an M-torsor (X, ϱ^X, β), define the coaction on M,

$$D(X, \varrho^X, \beta) := \beta^*(\varrho^X) = (\beta^{-1}\otimes_A\mathfrak{C}) \circ \varrho^X \circ \beta$$

(cf. Lemma 2.1). □

2.3. **Module-twisted forms.** For k-algebras A and B, fix a (B, A)-bimodule Σ and a right B-module N. A right B-module P is called a Σ*-twisted form of* N in case there exists a right A-module isomorphism $\phi : P\otimes_B\Sigma \to N\otimes_B\Sigma$. Σ-twisted forms (P, ϕ), (Q, ψ) of N are said to be *equivalent* if P and Q are isomorphic as right B-modules. The equivalence classes of Σ-twisted forms of N are denoted by $\mathrm{Twist}(\Sigma, N)$. $\mathrm{Twist}(\Sigma, N)$ is a pointed set with the class of $(N, N\otimes_B\Sigma)$ as a distinguished point

2.4. **Descent cohomology and the Galois-comodule twisted forms.** Recall that a right $\mathfrak{C}$-comodule Σ is called a *Galois comodule* or $(\mathfrak{C}, \Sigma)$ is called a *Galois coring* in case Σ is finitely generated and projective as an A-module and the evaluation map

$$\mathrm{Hom}^{\mathfrak{C}}(\Sigma, \mathfrak{C})\otimes_B\Sigma \to \mathfrak{C}, \qquad f\otimes s \mapsto f(s),$$

is an isomorphism of right $\mathfrak{C}$-comodules. Here and in the remainder of this subsection B is the endomorphism ring $B := \mathrm{End}^{\mathfrak{C}}(\Sigma)$.

Theorem 2.4. *Let Σ be a Galois right $\mathfrak{C}$-comodule that is faithfully flat as a left B-module. Then, for all right B-modules N, there is an isomorphism of pointed sets*

$$D^1(\mathfrak{C}, N\otimes_B\Sigma) \simeq \mathrm{Twist}(\Sigma, N),$$

where $N\otimes_B\Sigma$ is a comodule with the induced coaction $N\otimes_B\varrho^\Sigma$.

Proof. First recall that by the Galois comodule structure theorem [10, Theorem 3.2] (cf. [3, 18.27]) the functors $\mathrm{Hom}^{\mathfrak{C}}(\Sigma, -)$ and $-\otimes_B\Sigma$ are inverse equivalences between the categories of right $\mathfrak{C}$-comodules and right B-modules. Take any right B-module N. Note that $\mathrm{Tors}(N \otimes_A \Sigma)$ are $\mathfrak{C}$-comodule isomorphism classes of comodules which are isomorphic to $N\otimes_B\Sigma$ as right A-modules, while $\mathrm{Twist}(\Sigma, N)$ are isomorphism classes of B-modules P such that $P\otimes_B\Sigma$ is isomorphic to $N\otimes_B\Sigma$ as a right A-module (and hence as a $\mathfrak{C}$-comodule, with induced coactions). Since $-\otimes_B\Sigma$ is an equivalence, there is an isomorphism of pointed sets $\mathrm{Tors}(N \otimes_A \Sigma) \simeq \mathrm{Twist}(\Sigma, N)$, and the assertion follows by Proposition 2.3 □

2.5. **Beyond faithfully flat and finite Galois comodules.** Start with two algebras A and B, an A-coring $\mathfrak{C}$ and a (B,A)-bimodule Σ with a left B-linear right $\mathfrak{C}$-coaction $\varrho^\Sigma : \Sigma \to \Sigma\otimes_A\mathfrak{C}$. This defines the functor $-\otimes_B \Sigma : \mathbf{M}_B \to \mathbf{M}^\mathfrak{C}$, where $\mathbf{M}_B$ is the category of right B-modules and $\mathbf{M}^\mathfrak{C}$ is the category of right $\mathfrak{C}$-comodules. If this functor is an equivalence, then the same reasoning as in the proof of Theorem 2.4 yields an isomorphism of pointed sets

$$D^1(\mathfrak{C}, N\otimes_B\Sigma) \simeq \mathrm{Twist}(\Sigma, N),$$

where $N\otimes_B\Sigma$ is a comodule with the induced coaction $N\otimes_B\varrho^\Sigma$.

Let $B = \mathrm{End}^\mathfrak{C}(\Sigma)$. For Galois comodules which are finitely generated and projective as right A-modules but *are not* faithfully flat as B-modules sufficient conditions for $-\otimes_B\Sigma$ to be an equivalence are found in [1, Theorem 4.6]. In particular $D^1(\mathfrak{C}, N\otimes_B\Sigma) \simeq \mathrm{Twist}(\Sigma, N)$, for a *cleft bicomodule* Σ for a right coring extension $\mathfrak{D}$ of $\mathfrak{C}$ provided $\mathfrak{D}$ has a grouplike element (cf. [1, Definition 5.1, Corollary 5.5]). For Galois comodules in the sense of Wisbauer [15] (i.e. comodules Σ such that the map of functors $\mathrm{Hom}_A(\Sigma, -)\otimes_B\Sigma \to -\otimes_A\mathfrak{C}$, $\phi\otimes s \mapsto (\phi\otimes_A\mathfrak{C})(\varrho^\Sigma(s))$ is a natural isomorphism, no finiteness assumption on Σ_A) the functor $-\otimes_B\Sigma : \mathbf{M}_B \to \mathbf{M}^\mathfrak{C}$ is an equivalence if and only if the functor $-\otimes_B\Sigma : \mathbf{M}_B \to \mathbf{M}_A$ is comonadic (cf. [1, Section 1]).

Even more generally, Σ-twisted forms can be defined if the algebra B is non-unital (e.g. B can be an ideal in $\mathrm{End}^\mathfrak{C}(\Sigma)$). If B is *firm* in the sense that the product map $B \otimes_B B \to B$ is an isomorphism, and Σ is a firm B-module, then in the case of *infinite comatrix corings* the conditions for $-\otimes_B\Sigma$ to be an equivalence are given in [11, Theorem 5.9], [12, Theorem 4.15] (cf. [7, Theorem 1.3]).

2.6. **Comparison with the results of** [13]**.** To a given Hopf algebra H and a right H-comodule algebra A, one associates an A-coring $\mathfrak{C} := A\otimes_k H$ with the obvious left A-action, diagonal right A-action and the coproduct and counit $A\otimes_k\Delta_H$ and $A\otimes_k\varepsilon_H$ (cf. [3, 33.2]). The category of (A, H)-Hopf modules is isomorphic to the category of right $\mathfrak{C}$-comodules (cf. [2, Proposition 2.2]). Since A is a right H-comodule algebra, it is an (A, H)-Hopf module, hence a $\mathfrak{C}$-comodule. The endomorphism ring $\mathrm{End}^\mathfrak{C}(A)$ can be identified with the subalgebra of coinvariants $B := \{b \in A \mid \varrho^A(b) = b\otimes 1\}$. A is a Galois $\mathfrak{C}$-comodule if and only if $B \subseteq A$ is a Hopf-Galois H-extension. With these identifications in mind, the definition of the descent cohomology in [13] is a special case of Definition 2.2, [13, Lemma 2.2] can be derived from Lemma 2.1, [13, Theorem 2.6] follows by Theorem 2.4, while [13, Proposition 2.8] is a special case of Proposition 2.3.

2.7. **Galois cohomology for partial group actions.** As the theory of corings covers all known examples of Hopf-type modules, such as Yetter-Drinfeld modules, Doi-Koppinen Hopf modules, entwined and weak entwined modules, the results of the present note can be easily applied to all these special cases (cf. [3] for more details). One of the special cases of the descent cohomology for corings is that of partial Galois actions for non-commutative rings studied in [5] (cf. [9]). In this section we propose to use the descent cohomology to define the Galois cohomology for partial group actions, thus extending the classical Galois cohomology [14, Section I.5].

Take a finite group G and an algebra A. To any element $\sigma \in G$ associate a central idempotent $e_\sigma \in A$ and an isomorphism of ideals $\alpha_\sigma : Ae_{\sigma^{-1}} \to Ae_\sigma$. Following [5] we

say that the collection $(e_\sigma, \alpha_\sigma)_{\sigma\in G}$ is an *idempotent partial action* of G on A if $Ae_1 = A$, $\alpha_1 = A$ and, for all $a \in A$, $\sigma, \tau \in G$,

$$\alpha_\sigma(\alpha_\tau(ae_{\tau^{-1}})e_{\sigma^{-1}}) = \alpha_{\sigma\tau}(ae_{(\sigma\tau)^{-1}})e_\sigma,$$

(see [8, Definition 1.1] for the most general definition of a partial group action). Given an idempotent partial action $(e_\sigma, \alpha_\sigma)_{\sigma\in G}$ of G on A, define the invariant subalgebra of A,

$$A^G := \{a \in A \mid \forall \sigma \in G,\ \alpha_\sigma(ae_{\sigma^{-1}}) = ae_\sigma\}.$$

The extension $A^G \subseteq A$ is said to be *G-Galois* if and only if the map

$$A\otimes_{A^G} A \to \bigoplus_{\sigma\in G} Ae_\sigma, \qquad a \otimes a' \mapsto \sum_{\sigma\in G} a\alpha_\sigma(a'e_{\sigma^{-1}})v_\sigma$$

is bijective. Here v_σ denotes the element of $\bigoplus_{\sigma\in G} Ae_\sigma$ which is equal to e_σ at position σ and to zero elsewhere. For example, if there exists a convolution invertible right colinear map $k(G) \to A$, where $k(G)$ is the Hopf algebra of functions on G, then $A^G \subseteq A$ is a G-Galois extension known as a *cleft extension* [1, Section 6.5].

By [5, Proposition 2.2] (or as a matter of definition), $(e_\sigma, \alpha_\sigma)_{\sigma\in G}$ is an idempotent partial action of G on A if and only if $\mathfrak{C} := \bigoplus_{\sigma\in G} Ae_\sigma$ is an A-coring with the following A-actions, coproduct and counit:

$$a(a'v_\sigma)a'' = aa'\alpha_\sigma(a''e_{\sigma^{-1}})v_\sigma, \quad \Delta_{\mathfrak{C}}(av_\sigma) = \sum_{\tau\in G} av_\tau\otimes_A v_{\tau^{-1}\sigma}, \quad \varepsilon_{\mathfrak{C}}(av_\sigma) = a\delta_{\sigma,1},$$

for all $a, a', a'' \in A$ and $\sigma \in G$. Furthermore, the extension $A^G \subseteq A$ is G-Galois if and only if $\bigoplus_{\sigma\in G} Ae_\sigma$ is a Galois coring (with respect to the grouplike element $\sum_{\sigma\in G} v_\sigma$). Following [5, Definition 2.4], a right A-module M together with right A-module maps $(\varrho_\sigma : M \to M_\sigma)_{\sigma\in G}$ is called a *partial Galois descent datum*, provided $\varrho_1 = M$ and each of the ϱ_σ restricted to $Me_{\sigma^{-1}}$ is an isomorphism. In view of [5, Proposition 2.5], partial Galois descent data on a right A-module M are in bijective correspondence with descent cycles $Z^1(\bigoplus_{\sigma\in G} Ae_\sigma, M)$. In particular, for any right A^G-module N, there is a partial Galois descent datum

$$M = N\otimes_{A^G} A, \quad \varrho_\sigma : N\otimes_{A^G} A \to N\otimes_{A^G} Ae_\sigma, \quad n\otimes a \mapsto n\otimes\alpha_\sigma(ae_{\sigma^{-1}})e_\sigma.$$

This is simply the right $\mathfrak{C}$-coaction induced from the coaction on A given by the grouplike element $\sum_{\sigma\in G} v_\sigma$. A *partial Galois cohomology* of the G-Galois extension $A^G \subseteq A$ with values in the automorphism group $\mathrm{Aut}_A(M)$ of the right A-module $M = N\otimes_{A^G} A$ is defined as

$$H^i(G, \mathrm{Aut}_A(M)) := D^i(\bigoplus_{\sigma\in G} Ae_\sigma, M), \qquad i = 0, 1.$$

By Theorem 2.4, if A is a faithfully flat A^G-module, then $H^i(G, \mathrm{Aut}_A(M))$ describes equivalence classes of A-twisted forms of N.

As an example take $N = A^G$. In this case $M = A$ and the automorphism group $\mathrm{Aut}_A(A)$ can be identified with the group of units $\mathcal{G}(A)$ and $\mathrm{Aut}^{\mathfrak{C}}(A)$ can be identified with $\mathcal{G}(A^G)$. Furthermore, there is a bijective correspondence between right coactions of $\bigoplus_{\sigma\in G} Ae_\sigma$ on A and the set of grouplike elements in $\bigoplus_{\sigma\in G} Ae_\sigma$ (cf. [2, Lemma 5.1]).

In this way we obtain

$$H^0(G, \mathcal{G}(A)) = \mathcal{G}(A^G), \qquad H^1(G, \mathcal{G}(A)) = \mathcal{G}\left(\bigoplus_{\sigma\in G} Ae_\sigma\right)/\mathcal{G}(A),$$

where $\mathcal{G}(A)$ acts on A from the right by conjugation as in Section 1.2.

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